HALL EFFECTS ON MAGNETO HYDRODYNAMIC FLOW PAST AN EXPONENTIALLY ACCELERATED VERTICAL PLATE IN A ROTATING FLUID WITH MASS TRANSFER EFFECTS.

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ABSTRACT
The theoretical solution of flow past an exponentially accelerated vertical plate in the presence of Hall current and Magneto Hydrodynamic relative to a rotating fluid with uniform temperature and variable mass diffusion is presented. The dimensionless equations are solved using Laplace method. The axial and transverse velocity, temperature and concentration fields are studied for different parameters such as Hall parameter, Hartmann number, Rotation parameter, Schmidt number, Prandtl number, thermal Grashof number and mass Grashof number. It has been observed that the temperature of the plate decreases with increasing values of Prandtl number and the concentration near the plate increases with decreasing values of Schmidt number. It is also observed that Axial velocity increases with decreasing values of Magnetic field parameter, Hall parameter and Rotation parameter, whereas the transverse velocity increases with increasing values of Rotation parameter and Magnetic parameter but the trend gets reversed with respect to the Hall parameter. The effects of all parameters on the axial and transverse velocity profiles are shown graphically.

Index Terms: Hall Effect, MHD flow, Rotation, exponentially, accelerated plate, variable mass diffusion.

Nomenclature

$a, A, \alpha' \rightarrow$ Constants
$B_0 \rightarrow$ Applied Magnetic Field (T)
$C \rightarrow$ Dimensionless concentration
$c' \rightarrow$ Species concentration in the fluid (mol/m$^3$)
$c_p \rightarrow$ Specific heat at constant pressure (J/(kg.K))
$c_w' \rightarrow$ Concentration of the plate
$c_{\infty}' \rightarrow$ Concentration of the fluid far away from the plate
$D \rightarrow$ Mass diffusion co-efficient
$erfc \rightarrow$ Complementary error function

$Gc \rightarrow$ Mass Grashof number
$Gr \rightarrow$ Thermal Grashof number
$g \rightarrow$ Acceleration due to gravity (m/s$^2$)
$k \rightarrow$ Thermal conductivity (W/m.K)
$M \rightarrow$ Hartmann number
$m \rightarrow$ Hall parameter
$Pr \rightarrow$ Prandtl number
$Sc \rightarrow$ Schmidt number
$T \rightarrow$ Temperature of the fluid near the plate (K)
$T_w \rightarrow$ Temperature of the plate (K)
$T_{\infty} \rightarrow$ Temperature of the fluid far away from the plate (K)
$t \rightarrow$ Dimensionless time
$t' \rightarrow$ Time (s)
$u_0 \rightarrow$ Velocity of the plate
$(u', v', w') \rightarrow$ Components of Velocity Field F. (m/s)

$(u, v, w) \rightarrow$ Non-Dimensional Velocity Components
$(x', y', z') \rightarrow$ Cartesian Co-ordinates
$z \rightarrow$ Non-Dimensional co-ordinate normal to the plate.
$\mu \rightarrow$ Magnetic Permeability (H/m)
$\nu \rightarrow$ Kinematic Viscosity (m$^2$/s)

$\Omega' \rightarrow$ Component of Angular Velocity (rad/s)
$\Omega \rightarrow$ Non -Dimensional Angular Velocity
$\rho \rightarrow$ Fluid Density (kg/m$^3$)
$\sigma \rightarrow$ Electric Conductivity (Siemens/m)
$\theta \rightarrow$ Dimensionless temperature
$\eta \rightarrow$ Similarity parameter
$\beta \rightarrow$ Volumetric coefficient of thermal expansion
$\beta' \rightarrow$ Volumetric coefficient of expansion with concentration

I Introduction
Magneto Hydro Dynamics(MHD) flows of an electrically conducting fluid are encountered in many
industrial applications such as purification of molten metals, non-metallic intrusion, liquid metal, plasma studies, geothermal energy extraction, nuclear reactor and the boundary layer control in the field of aerodynamics and aeronautics.

The rotating flow of an electrically conducting fluid in the presence of magnetic fluid is encountered in cosmical, geophysical fluid dynamics. Also in solar physics involved in the sunspot development, the solar cycle and the structure of rotating magnetic stars. The study of MHD Viscous flows with Hall Currents has important engineering applications in problems of MHD Generators, Hall Accelerators as well as in Flight Magneto Hydrodynamics.

The effect of Hall currents on hydromagneto flow near an accelerated plate was studied by Pop.I (1971). Rotation effects on hydromagnetic free convective flow past an accelerated isothermal vertical plate was studied by Raptis and Singh(1981). H.S.Takhar.et.al., (1992) studied the Hall effects on heat and mass transfer flow with variable suction and heat generation. Watnab and pop(1995) studied the effect of Hall current on the steady MHD flow over a continuously moving plate, when the liquid is permeated by a uniform transverse magnetic field. Takhar et. al.(2002) investigated the simultaneous effects of Hall Current and free stream velocity on the magneto Hydrodynamic flow over a moving plate in a rotating fluid. Hayat and abbas (2007) studied the fluctuating rotating flow of a second-grade fluid past a porous plate with variable suction and Hall current. Muthucumaraswamy et. al.(2008) obtained the heat transfer effects on flow past an exponentially accelerated vertical plate with variable temperature. Magneto hydro dynamic convective flow past an accelerated isothermal vertical plate with variable mass diffusion was studied by Muthucumaraswamy.R.et.al., (2011).

In all the above studies, the combined effect of rotation and MHD flow in addition to Hall current has not been considered simultaneously. Here we have made an attempt to study the Hall current effects on a MHD flow of an exponentially accelerated horizontal plate relative to a rotating fluid with uniform temperature and variable mass diffusion.

II Mathematical Formulation
Here we consider an electrically conducting viscous incompressible fluid past an infinite plate occupying the plane $z' \leq 0$. The $x'$-axis is taken in the direction of the motion of the plate and $y'$-axis is normal to both $x'$ and $z'$-axes. Initially, the fluid and the plate rotate in unison with a uniform angular velocity $\Omega'$ about the $z'$-axis normal to the plate, also the temperature of the plate and concentration near the plate are assumed to be $T_\infty$ and $c'_\infty$. At time $t' > 0$, the plate is exponentially accelerated with a velocity $u' = \frac{u_0}{A} e^{(a't')}$ in its own plane along $x'$-axis and the temperature from the plate is raised to $T_w$, and the concentration level near the plate is raised linearly with time. Here the plate is electrically non conducting. Also, a uniform magnetic field $B_0$ is applied parallel to $z'$-axis. Also the pressure is uniform in the flow field. If $u', v', w'$ represent the components of the velocity vector $F$, then the equation of continuity $\nabla \cdot F = 0$ gives $w' = 0$ everywhere in the flow such that the boundary condition $w' = 0$ is satisfied at the plate. Here the flow quantities depend on $z'$ and $t'$ only and it is assumed that the flow far away from the plate is undisturbed. Under these assumptions the unsteady flow is governed by the following equations.

\[
\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial z'^2} + 2\Omega' v' - \frac{\sigma \mu^2 B_0^2}{\rho(1 + m^2)} (u' + mv') + g\beta(T - T_\infty) + g\beta * (c' - c'_\infty) \tag{1}
\]

\[
\frac{\partial v'}{\partial t'} = \nu \frac{\partial^2 v'}{\partial z'^2} - 2\Omega' u' + \frac{\sigma \mu^2 B_0^2}{\rho(1 + m^2)} (mu' - v') \tag{2}
\]

\[
\rho_c \frac{\partial T}{\partial t'} = K \frac{\partial^2 T}{\partial z'^2} \tag{3}
\]

\[
\frac{\partial c'}{\partial t'} = D \frac{\partial^2 c'}{\partial z'^2} \tag{4}
\]

Where $u'$ is the axial velocity and $v'$ is the transverse velocity. The prescribed initial and boundary conditions are

\[
u' = 0, \quad T = T_\infty, \quad c' = c'_\infty \quad \text{at} \quad t' \leq 0 \quad \text{for all} \quad z' \tag{5}
\]

\[
u' = \frac{u_0}{A} e^{(a't')}, \quad v' = 0, \quad T = T_w, \quad c' = c'_\infty + \Phi'_w - c'_\infty \frac{\partial T'}{\partial t'} \quad \text{at} \quad z' = 0 \quad \text{for all} \quad t' > 0 \tag{6}
\]

\[
u' \to 0, v' \to 0, T \to T_\infty, c' \to c'_\infty \quad \text{as} \quad z' \to \infty \tag{7}
\]
where, \( A = \left( \frac{u_0}{V} \right)^{\frac{1}{3}} \) is a constant.

On introducing the following non-dimensional quantities,

\[
\begin{align*}
  u &= \frac{u'}{\eta}, \\
  v &= \frac{v'}{\eta}, \\
  z &= z' \left( \frac{u_0}{V^2} \right), \\
  t &= \left( \frac{u_0}{V} \right)^{\frac{1}{3}} t', \\
  \Omega &= \Omega \left( \frac{V}{u_0} \right)^{\frac{1}{3}}, \\
  M^2 &= \frac{\sigma u_0^2 B_0^2 V^3}{2 \rho u_0^{2/3}}, \\
  a &= \left( \frac{\nu}{\eta} \right)^{\frac{1}{3}} a'. 
\end{align*}
\]

\[
G_c = \frac{8\beta'(c_w - c_{\infty})}{u_0}, \\
G_r = \frac{8\beta(T_w - T_{\infty})}{u_0}, \\
C = \frac{c' - c_{\infty}}{c_w - c_{\infty}}, \\
\theta &= \frac{T - T_{\infty}}{T_w - T_{\infty}}, \\
Pr &= \frac{\mu c_p}{K}, \\
Sc &= \frac{\nu}{D}
\]

The equations (1)-(7) reduce to the following non-dimensional form of governing equations

\[
\begin{align*}
  \frac{\partial u}{\partial t} &= \frac{\partial^2 u}{\partial \xi^2} + 2\Omega v + \frac{2M^2}{1 + m^2} \left( \eta + mv \right) + Gr \theta + GcC, \\
  \frac{\partial v}{\partial t} &= \frac{\partial^2 v}{\partial \xi^2} - 2\Omega u + \frac{2M^2}{1 + m^2} \left( \eta u - v \right), \\
  \frac{\partial \theta}{\partial t} &= \frac{1}{Pr} \frac{\partial^2 \theta}{\partial \xi^2}, \\
  \frac{\partial C}{\partial t} &= \frac{1}{Sc} \frac{\partial^2 C}{\partial \xi^2}
\end{align*}
\]

With initial and boundary conditions

\[
\begin{align*}
  u &= 0, \quad v = 0, \quad \theta = 0, \\
  C &= 0 \quad \text{at } t \leq 0 \quad \text{for all } z
\end{align*}
\]

\[
\begin{align*}
  u &= e^{at}, \quad v = 0, \\
  \theta &= 1, \quad C = t \quad \text{at } t > 0, \quad z = 0
\end{align*}
\]

\[
\begin{align*}
  u &\to 0, \quad v \to 0, \\
  \theta &\to 0, \quad C \to 0 \quad \text{as } z \to \infty
\end{align*}
\]

The above equations (8) - (9) and boundary conditions (12)-(14) can be combined as

\[
\begin{align*}
  \frac{\partial q}{\partial t} - \frac{\partial^2 q}{\partial \xi^2} &= -2q \left( \frac{M^2}{1 + m^2} + \left( \Omega - \frac{M^2m}{1 + m^2} \right) \right) + Gr \theta + GcC, \\
  \frac{\partial \theta}{\partial t} &= \frac{1}{Pr} \frac{\partial^2 \theta}{\partial \xi^2}, \\
  \frac{\partial C}{\partial t} &= \frac{1}{Sc} \frac{\partial^2 C}{\partial \xi^2}
\end{align*}
\]

With boundary conditions

\[
\begin{align*}
  q &= 0, \quad \theta = 0, \quad C = 0 \quad \text{at } t \leq 0 \quad \text{for all } z
\end{align*}
\]

\[
\begin{align*}
  q &= e^{at}, \quad \theta = 1, \quad C = t \quad \text{at } z = 0, \quad \text{for all } t > 0
\end{align*}
\]

\[
\begin{align*}
  q &\to 0, \quad \theta \to 0, \\
  C &\to 0 \quad \text{as } z \to \infty
\end{align*}
\]

Where \( q = u + iv \).

**III Solution of the problem.**

To solve the dimensionless governing equations (15) to (17), subject to the initial and boundary conditions (18)-(20) Laplace -Transform technique is used. The solutions are in terms of exponential and complementary error function:

\[
\begin{align*}
  c &= t \left[ \left( 1 + 2(1 + s)^2 \right) \text{erfc}(\eta \sqrt{s c}) \right] - \frac{2(1 + s)^2}{\sqrt{s}} \exp(\eta^2 sc) \\
  \theta &= \text{erfc}(\eta \sqrt{pr})
\end{align*}
\]
To interpret the results for a better understanding of the problem, numerical calculations are carried out for different physical parameters \( M, m, \Omega, Gr, Gc, Pr \) and \( Sc \). The value of Prandtl number is chosen to be 7.0 which correspond to water.

Figure 1 illustrates the effect of Schmidt number \( (Sc=0.16, 0.3, 0.6), M=m=0.5, \Omega =0.1, a=2.0, t=0.2 \) on the concentration field. It is observed that, as the Schmidt number increases, the concentration of the fluid medium decreases.

The effect of Prandtl number \( (Pr) \) on the temperature field is shown in Figure 2. It is noticed that an increase in the prandtl number leads to a decrease in the temperature.

IV Results and Discussion

In order to get a clear understanding of the flow field, we have separated \( q \) into real and imaginary parts to obtain axial and transverse components \( u \) and \( v \).

Where

\[
b = 2 \left[ \frac{M^2}{1+m^2} + i(\Omega - \frac{M^2 m}{1+m^2}) \right], \quad d_1 = \frac{Gr}{Pr-1}, \quad e_1 = \frac{b}{Pr-1},
\]

\[
d_2 = \frac{Gc}{Sc-1}, \quad e_2 = \frac{b}{Sc-1}, \quad \eta = z / 2 \sqrt{t}
\]

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The effect of Rotation parameter on axial velocity is shown in Figure 4. It is observed that the velocity increases with decreasing values of $\Omega$ (3.0, 5.0).

Fig. 5 demonstrates the effect of Hall parameter $m$ on axial velocity. It has been noticed that the velocity decreases with increasing values of Hall parameter.

Figures 6 and 7 show the effects of thermal Grashof number $Gr$ and mass Grashof number $Gc$. It has been noticed that the axial velocity increases with increasing values of both $Gr$ and $Gc$.

Figure 8 illustrates the effects of Magnetic field parameter $M$ on transverse velocity. It is observed that the transverse velocity increases with increasing values of $M$ and it is also observed that the transverse velocity peaks closer to the wall as $M$ is increased.
The Transverse velocity profiles for different values of Rotation parameter $\Omega$ are shown in Figure 9. It is observed that the velocity increases with decreasing values of $\Omega$.

Figure 10 shows the effect of Hall parameter $m$ on transverse velocity. It is found that the velocity increases with increasing values of $m$.

The effect of Mass Grashof number on transverse velocity is shown in Figure 12. Numerical calculations were carried out for different values of $Gc$ namely 3.0, 5.0, 10.0. From the Figure it has been noticed that with decreasing values of $Gc$ the transverse velocity is increased.
V Conclusion

In this paper we have studied the effects of Hall current, Rotation effect on MHD flow through an exponentially accelerated vertical plate with uniform temperature and variable mass diffusion. In the analysis of the flow the following conclusions are made.

(1) The concentration near the plate increases with decreasing values of the Schmidt number.
(2) The temperature of the plate decreases with increasing values of prandtl number
(3) Both Axial velocity and Transverse velocity increase with decreasing values of Magnetic field parameter or Rotation parameter. Also the Axial velocity increase with decreasing values of Hall parameter but the trend is reversed in Transverse velocity.
(4) Both Axial velocity and Transverse velocity increase with increasing values of thermal Grashof number.
(5) Axial velocity increases with increasing values of mass Grashof number whereas the transverse velocity increase with decreasing values of mass Grashof number.

References